

Time singularities of correlators from Dirichlet conditions in AdS/CFT

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Abstract

Within AdS/CFT, we establish a general procedure for obtaining the leading singularity of two-point correlators involving operator insertions at different times. The procedure obtained is applied to operators dual to a scalar field which satisfies Dirichlet boundary conditions on an arbitrary time-like surface in the bulk. We determine how the Dirichlet boundary conditions influence the singularity structure of the field theory correlation functions. New singularities appear at boundary points connected by null geodesics bouncing between the Dirichlet surface and the boundary. We propose that their appearance can be interpreted as due to a non-local double trace deformation of the dual field theory, in which the two insertions of the operator are separated in time. The procedure developed in this paper provides a technical tool which may prove useful in view of describing holographic thermalization using gravitational collapse in AdS space.

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1 Introduction

One of the central questions in the physics of heavy ion collisions is the mechanism of thermalization. A short thermalization time of the order of $0.5fm/c$ is suggested by successful hydrodynamical descriptions of elliptic flow data, yet its mechanism remains poorly understood. Recently, there have been increasing amount of efforts in the studies of thermalization with both phenomenological and theoretical approaches. Different scenarios have been proposed to describe the thermalization process. We will mention a few of them here: In [1], the role of quantum fluctuations were emphasized in the equilibration of initial coherent fields in a Color-Glass-Condensate approach. In [2], the mechanism driving the system to equilibrium was attributed to an interplay between the plasma instability and Bjorken expansion. A possible Bose-Einstein condensation of gluons at the onset of equilibration has also been proposed [3].

The abovementioned studies are all based on a weak coupling picture. Studies of thermalization in the strong coupling regime have been mainly carried out in the framework of the AdS/CFT correspondence. These include gravitational collapse models [4–9] as well as gravitational shock wave collision models [10–17]. While these works have been successful in describing the formation of a black hole in AdS, which corresponds to the equilibration of gauge fields, the evolution of correlation functions in the process of thermalization remains largely unexplored. Recently, there has been some initial progress in this direction [6,18–21]. In particular, [21] studied various quantities, including equal-time correlators, Wilson loops and entanglement entropy in a thermalization process of a strongly coupled gauge theory within a gravitational collapse model. Moreover, in [21] a “top-down” scenario was proposed that gives rise to a thermalization cascade from UV to IR modes for the dual strongly coupled gauge theory. Whether this cascade is a universal feature of strongly coupled theories with a holographic dual remains to be seen. This behaviour is in contrast to the “bottom-up” scenario established in weak coupling analysis [22], where the IR mode equilibrates first, followed by a loss of energy of the UV mode.

While the observables that have been studied previously contain valuable information about the state of the system at a given time, more information can be extracted from further observables that are not evaluated at fixed times, for instance from correlators involving operators at different times. In contrast to systems at thermal equilibrium, the response of the system is not determined by the fluctuation-dissipation theorem. Moreover, correlation of operators at different times depends not only on the time difference but also on the initial time. This kind of observables are particularly valuable when studying the

long-time behaviour of the system following an initial perturbation.

Interestingly, the thermalization mechanism has also been investigated recently in the condensed matter community, mainly due to the advancement of experimental tools for the study of ultracold atoms and quantum phase transitions [23]. A canonical question in condensed matter physics is how a system behaves under a quantum quench. This refers to the behavior of a system after a sudden change of a parameter in the original Hamiltonian, after which it undergoes a unitary evolution according to the new Hamiltonian. This can lead to thermalization. General expressions for one-point and two-point correlators after a quantum quench have been obtained for $d = 1$ space dimensions using two-dimensional CFT techniques in the seminal work [24]. Moreover, these authors argue that the two-point correlator has the same generic behavior also in higher dimensions. These novel results have triggered an extensive study of quantum quench also in higher-dimensional CFT by methods of the AdS/CFT correspondence, with many interesting results [25–28]. However, the mechanism of thermalization in higher dimension is still unclear, as opposed to its $(1 + 1)$ -dimensional counterpart.

In view of these results, in this paper we establish a new technical tool which we expect to be useful in further studies of thermalization and out-of equilibrium processes in general. We focus on a particular class of observables, i.e. the spatially integrated unequal-time correlator $\langle \int d^{d-1}x O(t, x) O(t', 0) \rangle$. We study the singularity structure of the correlator in the framework of AdS/CFT. To be specific, we consider the operator dual to a bulk scalar. Extensions to other operators are straightforward.

A standard thermalization model in AdS/CFT consists of a collapsing shell of matter in the bulk that eventually forms a black hole. In order to illustrate our method, here however we consider a simpler setup where the shell of matter has a completely reflective surface with Dirichlet boundary conditions - a mirror - and does not backreact on the geometry. We will not make assumptions about the evolution of the collapse, but we work out simple examples explicitly. Although this is a crude approximation to the real problem, it may capture some of the features of a real collapsing shell before the formation of a horizon starts. In particular, we obtain useful results on divergence matching which are expected to generalize to boundary conditions other than Dirichlet. We plan to apply these result to the collapsing shell geometry with appropriate boundary conditions in the future.

This work is based on the previous work [30] by Amado and one of the present authors and [31] by Ngo and the other two of the present authors. In [30], the scalar wave in AdS space with a static mirror, which provides a Dirichlet boundary condition,

has been considered. The corresponding two-point correlator on the boundary field theory showed singularities when the insertion times of the operators are connected a bouncing null geodesics between the mirror and the boundary. In [31], a mirror trajectory which breaks time translational invariance but preserves scaling invariance has been considered. An explicit evaluation of the two-point correlator in this moving mirror setting confirmed that the structure of the singularities in the correlator is consistent with a bouncing null geodesic picture. These two examples are realizations of the bulk-cone singularity conjecture [29] in Poincaré coordinates. According to this conjecture, singularities in boundary correlators appear whenever two insertion points at the boundary are connected by null geodesics in the bulk. This is based on the observation that bulk correlators are singular on null surfaces and those are inherited by the boundary correlators. Such singularities can appear inside the boundary lightcone, so they contain both information about the bulk that can be used to learn about its causal structure, and dynamical information about the boundary theory. This was applied to study the stages prior to horizon formation by a collapsing shell in the original work proposing the bulk-cone singularity conjecture [29].

In previous works the bulk-cone singularity was used to determine the location of time singularities of dual correlators. In the present work, we extend the analysis to determine the precise form of the singularities and their coefficients in the presence of an arbitrary moving mirror in AdS.

We expect that these results will provide an essential tool for the future study of the thermalization process in the gravitational collapse model. Knowledge of the singularity structure of correlators as described above will provide useful information about the behavior of strongly coupled gauge theories far away from equilibrium, in particular as far as decoherence is concerned.

This paper is organized as follows: In section 2, we present and sketch the derivation of the divergence matching method. We find a set of recursion equations relating the most singular part of the two-point correlator at adjacent singularities. The initial conditions to the recursion equations are obtained from the vacuum bulk-boundary propagator. Solving the recursion equations with the initial conditions allows us to determine the precise form for all singularities. In sections 3 and 4, we test of the divergence matching method with explicit evaluations of the two-point correlator in the cases of static mirror [30] and mirror in constant motion [31]. In section 5, we interpret the Dirichlet boundary condition as non-local double trace deformation in the dual field theory and argue that the non-local double trace deformation generically leads to the emergence of new singularities in the

two-point correlator. In section 6, the form of the singularities is explained in terms of spectral decomposition for the cases of the static mirror and the scaling mirror. We end with conclusion and outlook in section 7.

2 Singularities for arbitrary mirror trajectories

Our goal is to find the coefficient of singularities of the two-point correlator of a scalar operator, using holography and imposing a Dirichlet boundary condition on an arbitrary (time-like) surface $z = f(t) > 0$, for the dual scalar field in an AdS geometry

$$ds^2 = \frac{1}{z^2} (dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu).$$

In general, to solve the equations of motion with an arbitrary condition of this kind is too difficult, and only in some simple cases explicit solutions are known. However, in order to find the singularities it is not necessary to know the full solution, the leading terms in a WKB approximation are sufficient. As it has been shown in several examples, the localization of singularities can be obtained from null geodesics bouncing on the mirror [30, 31].

Here, in order to study the effect of the Dirichlet boundary condition, we introduce by hand a potential barrier localized at the position of the mirror in the equations of motion of the scalar field, and then we take the strength to infinity. For simplicity, we will focus on a massless scalar field. The equations of motion in the presence of the potential are

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu) \Phi + V \Phi = 0, \quad (1)$$

where $V = V_0 \delta(z - f(t))$. (1) is a Klein-Gordon equation in the presence of a potential. Experience from Quantum Mechanics tells us that the limit $V_0 \rightarrow \infty$ corresponds to the Dirichlet boundary condition at $z = f(t)$.^{#1}

The computation of the two-point correlator is equivalent to finding a bulk-boundary correlator, which satisfies the condition:

$$(\square + V)G(t, z, t') = 0, \text{ with } G(t, z \rightarrow 0, t') \rightarrow \delta(t - t'). \quad (2)$$

We restrict ourselves to spatially homogeneous solutions to (2), which will lead to the spatially integrated two-point correlator [31]. With this simplification, the Laplacian operator in AdS_{d+1} reduces to $\square = -z^2 \partial_t^2 + z^2 \partial_z^2 + z(1-d) \partial_z$. (2) can be reformulated as the integral equation

$$G(t, z, t') = G_0(t, z, t') - \int G_{bb}(t, z, t'', z'') V(t'', z'') \sqrt{-g(z'')} G(t'', z'', t') dt'' dz'', \quad (3)$$

^{#1}Examples can be found in Appendices B and C.

where $G_0(t, z, t')$ and $G_{bb}(t, z, t'', z'')$ are the bulk-boundary propagator and the bulk-bulk propagator in the absence of a potential. They are defined as follows:

$$\begin{aligned} \square G_0(t, z \rightarrow 0, t') &= 0 \\ \text{with } G_0(t, z \rightarrow 0, t') &\rightarrow \delta(t - t') , \end{aligned} \quad (4)$$

$$\begin{aligned} \square G_{bb}(t, z, t', z') &= \frac{1}{\sqrt{-g}} \delta(t - t'') \delta(z - z'') \\ \text{with } G_{bb}(t, z \rightarrow 0, t', z') &\rightarrow 0. \end{aligned} \quad (5)$$

Let us focus on the time-ordered propagators. The explicit expressions are given by [32]:

$$G_0(t, z, t') = \frac{i}{\pi} \frac{\Gamma(\frac{d+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{d}{2})} \frac{z^d}{(-(t - t')^2 + z^2 + i\epsilon)^{\frac{d+1}{2}}} , \quad (6)$$

$$G_{bb}(t, z, t', z') = -\frac{i}{2\pi} (zz')^{\frac{d-1}{2}} Q_{\frac{d-1}{2}} \left(\frac{z^2 + z'^2 - (t - t')^2 + i\epsilon}{2zz'} \right). \quad (7)$$

Note the $i\epsilon$ prescription is chosen for time ordered correlators (Feynman). For $t > t'$, $G_0(t, z, t')$ contains only contribution from positive frequency modes, and for $t < t'$, $G_0(t, z, t')$ contains only contribution from negative frequency modes. Similarly, for $t > t'$, $G_{bb}(t, z, t', z')$ is a propagator for positive frequency modes, and for $t < t'$, $G_{bb}(t, z, t', z')$ is a propagator for negative frequency modes. These properties will be crucial in the analysis below. The delta function in the potential V forces the integration in (3) to be performed along the mirror trajectory.

Notice the following: G_0 and G_{bb} are independent of V_0 , so in order for (3) to be consistent in the $V_0 \rightarrow \infty$ limit, two conditions must be satisfied, the first is that $G \sim 1/V_0$ and the second is that the $O(V_0^0)$ contributions from G_0 and the integral term cancel out.^{#2} From the last condition one can obtain the coefficients of singularities. The idea works as follows: Let us define $G = \frac{1}{V_0} G_{LO} + O(V_0^{-2})$, then

$$\begin{aligned} &\int G_{bb}(t, z, t'', z'') V(t'', z'') G(t'', z'', t') \sqrt{-g(z'')} dt'' dz'' \\ &= \int G_{bb}(t, z, t'', f(t'')) G_{LO}(t'', f(t''), t') \sqrt{-g(f(t''))} dt'' + O(V_0^{-1}). \end{aligned} \quad (8)$$

Then, to the order $O(V_0^0)$, we have the following conditions

$$G_0(t, z, t') - \int G_{bb}(t, z, t'', f(t'')) G_{LO}(t'', f(t''), t') \sqrt{-g(f(t''))} dt'' = 0. \quad (9)$$

We first look at (8) along the trajectory of the mirror: $z = f(t)$, Generically, we expect $G_{LO}(t'', f(t''), t')$ to be singular whenever t'' is connected by a null geodesic to t' , $t'' \rightarrow t_n$,

^{#2}Then, in this limit we should normalize the correlator by multiplying it by a factor of V_0 in order to obtain a finite result.

as remarked in Fig.1. Furthermore, G_{bb} has a singularity whenever the points $(t, f(t))$ and $(t'', f(t''))$ are connected by a null geodesic bouncing once at the boundary. One can see this from (6), the function $Q_{\frac{d-1}{2}}(x)$ has logarithmic singularities at $x = \pm 1$. This gives the condition

$$f(t)^2 + f(t'')^2 - (t - t'')^2 = \pm 2f(t)f(t'') \Rightarrow (f(t) \mp f(t''))^2 = (t - t'')^2. \quad (10)$$

Singularities appear at $t = t''$ and $t = t'' \pm \Delta t$, where $\Delta t = f(t) + f(t'')$ is the time it takes a null ray to go from $z = f(t)$ to $z'' = f(t'')$ bouncing once at the boundary. e.g., when (8) is evaluated at $(t_0, f(t_0))$ the term in the integral has a singular contribution from $(t_{-1}, f(t_{-1}))$, $(t_0, f(t_0))$ and from $(t_1, f(t_1))$, the next point in the mirror connected by a null geodesic bouncing at the boundary. The convolution of $G_{bb}(t, f(t), t'', f(t''))$ and $G_{LO}(t'', f(t''), t')$ close to $t'' \rightarrow t_n$ gives the most singular part, which is to be cancelled by the most singular part of $G_0(t, f(t), t')$. Note that G_0 is singular only when $t \rightarrow t_0, t_{-1}$. We arrive at the following matching conditions:

$$\sum_{n=m-1}^{m+1} \int^{t_n} K(t \rightarrow t_m, t'') G_{LO}(t'', t') dt'' \stackrel{\triangle}{=} 0, \quad m \neq 0, -1, \quad (11a)$$

$$G_0(t \rightarrow t_0, f(t), t') - \sum_{n=-1}^1 \int^{t_n} K(t \rightarrow t_0, t'') G_{LO}(t'', t') dt'' \stackrel{\triangle}{=} 0, \quad m = 0, \quad (11b)$$

$$G_0(t \rightarrow t_{-1}, f(t), t') - \sum_{n=-2}^0 \int^{t_n} K(t \rightarrow t_{-1}, t'') G_{LO}(t'', t') dt'' \stackrel{\triangle}{=} 0, \quad m = -1, \quad (11c)$$

where $K(t, t'') = G_{bb}(t, f(t), t'', f(t'')) \sqrt{-g(f(t''))}$. The symbol $\stackrel{\triangle}{=}$ means the equality holds as far as the most singular part is concerned. The superscript of the integration sign means that the main contribution to the integral comes from the singular behavior close to the singularities t_n . Focusing on the most singular part allows us to identify a set of discrete conditions, thus significantly simplifying the problem. We will illustrate how this works with the explicit examples of a static mirror [30] and a mirror with scaling trajectory [31].

Schematically, the singularities in $G_0(t, f(t), t')$ get propagated through (11a)-(11c) to $G_{LO}(t'', f(t''), t')$. Since we focus on the time-ordered the propagator, G_{LO} contains only positive (negative) frequency modes for $t'' > t'$ ($t'' < t'$). Therefore, the most singular part of G_{LO} assumes a similar form as G_0 :

$$G_{LO}(t, t') = G_{LO}(t = t_m(1 + y), z = f(t), t') \stackrel{\triangle}{=} \begin{cases} \frac{g_m}{(-iy + \epsilon)^{c+1}} & m \leq -1 \\ \frac{g_m}{(iy + \epsilon)^{c+1}} & m \geq 0 \end{cases}, \quad (12)$$

where $c = \frac{d+1}{2}$ and g_m are some constants to be fixed by the matching procedure.

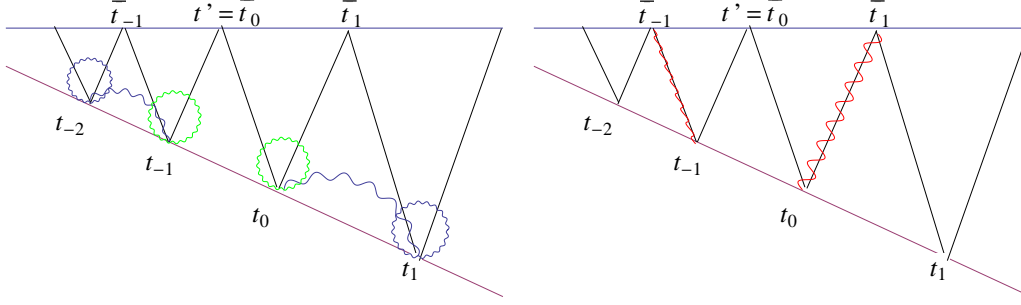


Figure 1: (color online) The figure on the left illustrates the cancellation of the most singular parts. The wavy lines denote the $K(t \rightarrow t_m, t'' \rightarrow t_n)$. For $m > 0$, the contributions from the blue wavy loop and the blue arc on the right cancel against each other, corresponding to $n = m$ and $n = m - 1$. For $m < 0$, the contributions from the blue wavy loop and the blue arc on the left cancel, corresponding to $n = m$ and $n = m + 1$. For $m = 0, -1$, only $n = m$ contributes. These contributions are depicted as green wavy loops, and cancel the most singular part of $G_0(t, z, t')$. The figure on the right illustrates how the most singular parts are propagated to the boundary: For $m > 0$ ($m < 0$), only $n = m - 1$ ($n = m$) contributes to the two-point correlator, as indicated by the red wavy line on the right (left).

Now we wish to determine g_m through the recursion relations (11a)-(11c). Close to the points where null geodesics bounce on the mirror $t = t_m(1 + x)$, $z \simeq f(t_m) + t_m f'(t_m)x$ and $t'' = t_n(1 + y)$, $z'' \simeq f(t_n) + t_n f'(t_n)y$, with $x, y \ll 1$, the most singular part of $K(t, t'')$ is given by

$$K(t = t_m(1 + x), t'' = t_n(1 + y)) \stackrel{d}{=} -\frac{i}{2\pi} \frac{z_m^{\frac{d-1}{2}}}{z_n} \frac{1}{z_n^2} \times \begin{cases} A_4 e^{\frac{-i\pi(d+1)}{2}} \ln \left(-\frac{1+f'(t_m)}{1-f'(t_n)} \frac{t_m}{t_n} x + y - i\epsilon \right) & n = m + 1 \\ A_4 e^{\frac{-i\pi(d+1)}{2}} \ln \left(\frac{1-f'(t_m)}{1+f'(t_n)} \frac{t_m}{t_n} x - y - i\epsilon \right) & n = m - 1 \\ A_4 \ln \left(-(x - y)^2 + i\epsilon \right) & n = m \end{cases}, \quad (13)$$

where we have defined $z_m = f(t_m)$, $z_n = f(t_n)$ and $A_4 = -\frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{2^{\frac{d+1}{2}}\Gamma(\frac{d+1}{4})\Gamma(\frac{d+3}{4})}$. Inserting (12) and (13) into (11a), and using the integrals in Appendix A, we find that for $m > 0$, only $n = m, m - 1$ contribute, while for $m < -1$, only $n = m, m + 1$ contribute. The physical reason for this is that $K(t \rightarrow t_m, t'' \rightarrow t_n)$ at $n = m - 1$ and $n = m + 1$ are propagators for positive or negative frequency modes only. The convolution of a positive (negative) propagator with a singularity due to negative (positive) frequency modes always gives a vanishing result. On the other hand, $K(t \rightarrow t_m, t'' \rightarrow t_n)$ at $n = m$ is a propagator for

both positive and negative frequency modes, thus always contributes to the integral. This mechanism is precisely encoded in the vanishing of the integrals in Appendix A. We have indicated the cancellation of the nonvanishing contributions in the left panel of Fig.1. As a result, we can derive the following relations among the g_m defined in (12):

$$\begin{aligned} e^{i\pi c}(1 - f'(t_m))^c \left(\frac{t_m}{z_m}\right)^{c+1} g_m + (1 + f'(t_{m-1}))^c \left(\frac{t_{m-1}}{z_{m-1}}\right)^{c+1} g_{m-1} &= 0, \quad m > 0, \\ (1 - f'(t_m))^c \left(\frac{t_m}{z_m}\right)^{c+1} g_m + e^{i\pi c}(1 + f'(t_{m-1}))^c \left(\frac{t_{m-1}}{z_{m-1}}\right)^{c+1} g_{m-1} &= 0, \quad m \leq -1, \end{aligned} \quad (14)$$

The initial condition for (14) follows from (9). Note that the singular behavior of G_0 is given by

$$G_0(t = t_m(1+x), t') \stackrel{d}{=} \frac{i}{\pi} \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\Gamma(\frac{d}{2})} \frac{z_m^d}{(2t_m z_m)^c} \begin{cases} (1 - f'(t_m))^{-c}(-x + i\epsilon)^{-c} & m = 0 \\ (1 + f'(t_m))^{-c}(x + i\epsilon)^{-c} & m = -1 \end{cases}. \quad (15)$$

Using (12), (13) and (15), as well as the integrals in Appendix A, we find that only $n = m$ contributes to the integral. The physical reason is again that the singularities due to positive and negative frequency modes cannot meet the correct propagators for the cases $n = m \pm 1$. As a result, we can easily obtain g_0 and g_{-1} as follows:

$$g_0 = \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{(2i)^c A_4} \left(\frac{z_0}{t_0}\right)^{c+1} (1 - f'(t_0))^{-c} \quad (16)$$

$$g_{-1} = \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{(2i)^c A_4} \left(\frac{z_{-1}}{t_{-1}}\right)^{c+1} (1 + f'(t_{-1}))^{-c}, \quad (17)$$

where we have used $i = e^{\frac{i\pi}{2}}$ to simplify the notation. To obtain the two point-correlator in the gauge theory, we need to propagate singularities at $t = t_n$ to the boundary, where they will appear at different times $t = \bar{t}_n$. To this end, we need to look at (3) in the limit $z \rightarrow 0$. Note that the time-ordered correlator $\langle TO(t)O(t') \rangle = \lim_{z \rightarrow 0} \frac{G(t, z \rightarrow 0, t')}{z^d}$. The most singular part of the two-point correlator is given by:

$$\langle TO(t)O(t') \rangle|_{t \rightarrow \bar{t}_n} \stackrel{d}{=} - \sum_{n=m-1}^m \int_{\bar{t}_n}^{t_n} \bar{K}(t \rightarrow \bar{t}_n, t'') G_{LO}(t'', t') dt'', \quad (18)$$

where

$$\bar{K}(t, t'') = \lim_{z \rightarrow 0} \frac{G_{bb}(t, z, t'', f(t''))}{z^d} \sqrt{-g(f(t''))}. \quad (19)$$

It is not difficult to work out the most singular part of $\bar{K}(t, t'')$ for an arbitrary mirror

trajectory:

$$\bar{K}(t = \bar{t}_m(1+x), t'' = t_n(1+y)) \stackrel{d}{=} -\frac{i}{\sqrt{\pi}(2z_n)^{c+1}} \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d+2}{2})} \times \begin{cases} (\bar{t}_m x - (1 - f'(t_n))t_n y + i\epsilon)^{-c} & n = m \\ (-\bar{t}_m x + (1 + f'(t_n))t_n y + i\epsilon)^{-c} & n = m - 1 \end{cases}. \quad (20)$$

Finally, inserting (12) and (20) into (18) and using the integrals in Appendix A, we obtain the most singular part of the two-point correlators. A similar mechanism is at work again: Time-ordered correlators only propagate singularities due to either positive or negative frequency modes, thus only one out of two contributions survives, which is indicated in the right panel of Fig. 1. We collect the final results for the most singular part of the two-point correlator in the following:

$$\begin{aligned} \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} \frac{\alpha_{m-1}g_{m-1}}{(-x+i\epsilon)^{2c}}, \quad m > 0, \\ \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} \frac{\beta_m g_m}{(x+i\epsilon)^{2c}}, \quad m < 0, \end{aligned} \quad (21)$$

where

$$\alpha_{m-1} = -\frac{i^{c+1}}{\sqrt{\pi}} \sin(\pi c) B(2c, 1-c) \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d+2}{2})} \left(\frac{t_{m-1}}{z_{m-1}}\right)^{c+1} \frac{(1+f'(t_{m-1}))^c}{(2\bar{t}_m^2)^c}, \quad m > 0, \quad (22)$$

$$\beta_m = -\frac{i^{c+1}}{\sqrt{\pi}} \sin(\pi c) B(2c, 1-c) \frac{\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d+2}{2})} \left(\frac{t_m}{z_m}\right)^{c+1} \frac{(1-f'(t_m))^c}{(2\bar{t}_m^2)^c}, \quad m < 0. \quad (23)$$

Although these expressions look complicated, we can easily extract interesting physics by comparing the ratio between the residues of two consecutive singularities. To this end, we normalize the two-point correlators (21) as:

$$\begin{aligned} \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} \frac{\alpha_{m-1}g_{m-1}\bar{t}_m^{2c}}{(-t + \bar{t}_m + i\epsilon)^{2c}}, \quad m > 0, \\ \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} \frac{\beta_m g_m \bar{t}_m^{2c}}{(t - \bar{t}_m + i\epsilon)^{2c}}, \quad m < 0. \end{aligned} \quad (24)$$

The ratio of consecutive residues are given by:

$$R_m \equiv \begin{cases} \frac{\alpha_m g_m \bar{t}_{m+1}^{2c}}{\alpha_{m-1} g_{m-1} \bar{t}_m^{2c}} = -e^{-i\pi c} \left(\frac{1+f'(t_m)}{1-f'(t_m)}\right)^c, & m > 0, \\ \frac{\beta_m g_m \bar{t}_m^{2c}}{\beta_{m-1} g_{m-1} \bar{t}_{m-1}^{2c}} = -e^{-i\pi c} \left(\frac{1+f'(t_{m-1})}{1-f'(t_{m-1})}\right)^c, & m < 0 \end{cases}. \quad (25)$$

If one introduces a signal at the boundary it will have replicas at the time \bar{t}_m . The strength of the replicas depends on the relative value of the residues at future singularities: It becomes stronger when $|R_m| > 1$ and weaker when $|R_m| < 1$.

3 Simple example: static mirror

Let us see how the procedure works in the case of a static mirror, as first studied in [30]. The mirror sits at a constant radial coordinate: $f(t) = z_s$. It is straightforward to determine the singular points from geometric optics:

$$t_m = t' + (2m + 1)z_s, \quad z_m = z_s \quad (26)$$

$$\bar{t}_m = t' + 2mz_s. \quad (27)$$

Since $f'(t) = 0$, (25) predicts that the signal is always reflected with the same amplitude for a static mirror:

$$|R_m| \equiv 1, \quad m > 0. \quad (28)$$

Note that along the mirror trajectory $G(t, z = f(t), t') \sim \frac{1}{V_0}$. The left hand side of (3) is $\frac{1}{V_0}$ suppressed as compared to the right hand side (rhs) in the limit $V_0 \rightarrow \infty$, which allows us to focus on the rhs to leading order (LO) in V_0 . Along the mirror trajectory, $G(t, z = f(t), t')$ is most singular when $t \rightarrow t_m$ and $K(t, z, t'')$ is logarithmically divergent when t and t'' are adjacent points on the mirror trajectory hit by the light ray, i.e. $t'' \rightarrow t_n (n = m - 1, m, m + 1)$, or when t and t'' coincide. As a result, (3) can be written as:

$$G_0(t \rightarrow t_m, z, t') - \sum_{n=m-1}^{m+1} \int^{t_n} K(t \rightarrow t_m, z, t'') G_{LO}(t'', t') dt'' \stackrel{d}{=} 0. \quad (29)$$

To proceed on, let us assume the most singular part of $G_{LO}(t, z, t')$ takes the following form along the mirror trajectory:

$$G_{LO}(t = t_m(1 + y), t') \stackrel{d}{=} \begin{cases} \frac{g_m}{(-iy + \epsilon)^{c+1}}, & m \leq -1, \\ \frac{g_m}{(iy + \epsilon)^{c+1}}, & m \geq 0. \end{cases} \quad (30)$$

The difference in $i\epsilon$ prescription indicates that the singularities are due to positive and negative frequency modes, respectively. As noted in the previous section, singularities from positive frequency contribute to the integral only when $n = m, m - 1$, while singularities from negative frequency contribute to the integral only when $n = m, m + 1$. We are now ready to insert the singular forms of $G(t \rightarrow t_m, t')$ and $K(t \rightarrow t_m, t'' \rightarrow t_n)$ from (30) and (13) into (29). We obtain the recursion relations among g_m in (14) as well as the initial conditions g_0 and g_{-1} in (16). For the case of a static mirror, we can easily solve for the coefficients and obtain

$$\begin{aligned} g_m &= \left(\frac{-1}{e^{i\pi c}} \right)^m \left(\frac{t_0}{t_m} \right)^{c+1} g_0, & m > 0, \\ g_m &= \left(\frac{-1}{e^{i\pi c}} \right)^{-1-m} \left(\frac{t_{-1}}{t_m} \right)^{c+1} g_{-1}, & m < -1, \end{aligned} \quad (31)$$

with the explicit initial conditions

$$\begin{aligned} g_0 &= \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{z_s^{c+1}}{(2i)^c t_0^{c+1}}, \\ g_{-1} &= \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{z_s^{c+1}}{(2i)^c t_{-1}^{c+1}}. \end{aligned} \quad (32)$$

To calculate the two-point correlator, we take the $z \rightarrow 0$ limit in (3), close to the boundary. Note the two-point correlator is simply

$$\langle TO(t)O(t') \rangle = \lim_{z \rightarrow 0} \frac{G(t, z, t')}{z^d}. \quad (33)$$

The most singular part of the two-point correlator can be readily obtained from (3),

$$\langle TO(t)O(t') \rangle \stackrel{d}{=} - \int \bar{K}(t, t'') G(t'', t') dt''. \quad (34)$$

We have dropped the G_0 term since it is the vacuum piece of the correlator, which only has a trivial lightcone singularity at $t = t'$. With the most singular part of $G_{LO}(t \rightarrow t_m, t'')$ obtained above and noting that $\bar{K}(t, t'')$ is most singular when t and t'' are connected by light ray (null geodesics), we can write (34) as

$$\langle TO(t \rightarrow \bar{t}_m)O(t') \rangle \stackrel{d}{=} - \sum_{n=m-1}^m \int^{t_n} \bar{K}(t \rightarrow \bar{t}_m, t'') G(t'', t') dt'' \quad m \neq 0. \quad (35)$$

Using the singular forms of $G_{LO}(t'' \rightarrow t_n, t')$ and $\bar{K}(t \rightarrow \bar{t}_m, t'' \rightarrow t_n)$ and taking into account that only one out of the two terms contributes, we readily reduce (35) to (21). Inserting (31) and (32) into (21), we finally obtain the most singular part of the correlator for the state defined by the static mirror:

$$\begin{aligned} \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} - \frac{i}{2\pi} \frac{\sqrt{\pi}\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d+2}{2})} \frac{1}{(2\bar{t}_m^2)^c} (e^{-i\pi c} - e^{i\pi c}) \\ &\times i^{c+1} B(2c, 1-c) \left(\frac{-1}{e^{i\pi c}} \right)^{m-1} \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{1}{(2i)^c (-x+i\epsilon)^{2c}}, \quad m > 0, \\ \langle TO(t = \bar{t}_m(1+x))O(t') \rangle &\stackrel{d}{=} - \frac{i}{2\pi} \frac{\sqrt{\pi}\Gamma(\frac{d+1}{2})}{\Gamma(\frac{d+2}{2})} \frac{1}{(2\bar{t}_m^2)^c} (e^{-i\pi c} - e^{i\pi c}) \\ &\times i^{c+1} B(2c, 1-c) \left(\frac{-1}{e^{i\pi c}} \right)^{-m-1} \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{1}{(2i)^c (x+i\epsilon)^{2c}}, \quad m < 0. \end{aligned} \quad (36)$$

We can cross-check the above results by direct evaluation of G_{LO} and the two-point correlator $\langle TO(t)O(t') \rangle$. We have done this in Appendix B and find perfect agreement!

4 Non-trivial example: mirror along scaling trajectory

In this section, we will test our procedure with a non-trivial example. A non-trivial yet still analytically tractable example was studied in [31]. It corresponds to a mirror with scaling trajectory $z = \frac{t}{u_0}$ with $u_0 > 1$. The trajectory breaks the translational symmetry in time, but preserves the scaling symmetry.

As for the static mirror, the location of singular points can be deduced from geometric optics:

$$t_m = t' \frac{u_0}{u_0 - 1} \left(\frac{u_0 + 1}{u_0 - 1} \right)^m, \quad (37)$$

$$\bar{t}_m = t' \left(\frac{u_0 + 1}{u_0 - 1} \right)^m. \quad (38)$$

Using (25) we find the ratio of consecutive residues is given by:

$$|R_m| = \left(\frac{u_0 + 1}{u_0 - 1} \right)^c. \quad (39)$$

So the residues grow when the mirror moves away from the boundary. We will comment more on this in Section 6.

Introducing now the expression for the trajectory $f(t) = \frac{t}{u_0}$ in (11a)-(11c), we obtain the recursion equations

$$e^{i\pi c}(u_0 - 1)^c g_m + (u_0 + 1)^c g_{m-1} = 0 \quad m > 0 \quad (40)$$

$$e^{i\pi c}(u_0 + 1)^c g_m + (u_0 - 1)^c g_{m+1} = 0 \quad m < -1, \quad (41)$$

together with the initial conditions

$$g_0 = \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{1}{(2i)^c u_0 (u_0 - 1)^c} \quad (42)$$

$$g_{-1} = \frac{\Gamma(\frac{d+1}{2})\Gamma(\frac{1}{2})}{\pi\Gamma(\frac{d}{2})} \frac{c}{A_4} \frac{1}{(2i)^c u_0 (u_0 + 1)^c}. \quad (43)$$

Solving the recursion equations, we obtain

$$g_m = g_0 (-1)^m e^{-im\pi c} \left(\frac{u_0 + 1}{u_0 - 1} \right)^{mc} \quad m > 0 \quad (44)$$

$$g_m = g_{-1} (-1)^{m+1} e^{i(m+1)\pi c} \left(\frac{u_0 + 1}{u_0 - 1} \right)^{(m+1)c} \quad m < -1. \quad (45)$$

The coefficients of the most singular part of (44) can be compared with the results from direct evaluation of $G_{LO}(t = t_m(1 + x), t')$. The procedure is analogous to that in the

previous section, with details included in Appendix C. Here we simply quote the results for $G_{LO}(t, x')$:

$$G_{LO}(t < t') \stackrel{d}{=} A_2 e^{\frac{i\pi(d-1)m}{2}} \frac{1}{(-iy + \epsilon)^{c+1}}, \quad n < 0, \quad (46)$$

$$G_{LO}(t > t') \stackrel{d}{=} A_2 e^{\frac{-i\pi(d-1)(m+1)}{2}} \frac{1}{(iy + \epsilon)^{c+1}}, \quad n \geq 0, \quad (47)$$

with $A_2 = 2^c \frac{\Gamma(c)}{\Gamma(d)} t'^{-c} \left(\frac{t_m}{u_0} \right)^{\frac{d-1}{2}} \frac{\Gamma(1+c)}{2\pi} i^c$.

The two results do not agree with each other at a first glance. The disagreement can be traced back to the difference in the potential used in two approaches. In the divergence matching procedure, we used the potential $V_1 = V_0 \delta(z - \frac{t}{u_0})$, while in Appendix C, we use $V_2 = V_0 \delta(\frac{t}{z} - u_0)$. The latter form of the potential is necessary in order to keep the scale invariance of theory.

To compare the two results on equal footing, we note that as $t'' \rightarrow t_m$,

$$V_2(t'', z'') = \frac{t_m}{u_0^2} V_1(t'', z''). \quad (48)$$

Using (48) to convert one potential into the other, and using that $t_m = t' \frac{u_0}{u_0-1} \left(\frac{u_0+1}{u_0-1} \right)^m$, we can show that the two results on the most singular part of G_{LO} along the mirror trajectory do agree with each other.

We may further compare the final results for the two-point correlator obtained from the two different approaches. This time the difference in the form of the potential does not cause a problem because the limit $V_0 \rightarrow \infty$ for both V_1 and V_2 gives the Dirichlet boundary condition along the trajectory of the mirror. The two-point correlator from the divergence matching procedure is obtained by inserting (44) to (21), which gives:

$$\begin{aligned} & \langle TO(t = \bar{t}_m(1+x)) O(t') \rangle \\ & \stackrel{d}{=} - \frac{i\Gamma(c)}{\sqrt{\pi}\Gamma(c + \frac{1}{2})} \left(\frac{1}{\bar{t}_m t'} \right)^c \frac{\Gamma(c)}{\Gamma(d)} \frac{\Gamma(1+c)}{2\pi} e^{-i\pi(c-1)(m-1)} \sin \pi c B(2c, 1-c) \frac{1}{(-x + i\epsilon)^{2c}}, \quad m \geq 0, \end{aligned} \quad (49)$$

$$\begin{aligned} & \langle TO(t = \bar{t}_m(1+x)) O(t') \rangle \\ & \stackrel{d}{=} - \frac{i\Gamma(c)}{\sqrt{\pi}\Gamma(c + \frac{1}{2})} \left(\frac{1}{\bar{t}_m t'} \right)^c \frac{\Gamma(c)}{\Gamma(d)} \frac{\Gamma(1+c)}{2\pi} e^{i\pi(c-1)(m+1)} \sin \pi c B(2c, 1-c) \frac{1}{(-x + i\epsilon)^{2c}}, \quad m \leq -1. \end{aligned} \quad (50)$$

The result of the two-point correlator from direct evaluation in Appendix C is:

$$\begin{aligned} & \langle TO(t = \bar{t}_m(1+x))O(t') \rangle \\ & \stackrel{d}{=} (\bar{t}_m t')^{-\frac{d+1}{2}} 2^d (1 - e^{i\pi d}) \frac{\Gamma(-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{1-d}{2})\Gamma(d)} e^{\frac{-i\pi(d-1)m}{2}} \frac{\Gamma(1+d)}{e^{-i\pi(d+1)}(-x+i\epsilon)^{d+1}2i\pi}, \quad m \geq 0, \end{aligned} \quad (51)$$

$$\begin{aligned} & \langle TO(t = \bar{t}_m(1+x))O(t') \rangle \\ & \stackrel{d}{=} (\bar{t}_m t')^{-\frac{d+1}{2}} 2^d (1 - e^{-i\pi d}) \frac{\Gamma(-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{1-d}{2})\Gamma(d)} e^{\frac{i\pi(d-1)m}{2}} \frac{\Gamma(1+d)}{e^{-i\pi(d+1)}(x+i\epsilon)^{d+1}2i\pi}, \quad m \leq -1. \end{aligned} \quad (52)$$

We show in Appendix C that the two approaches give exactly the same results!

5 Non-local double trace deformation

It has been established in [33] that a bulk Dirichlet boundary condition leads to a double trace deformation on the field theory, based on earlier works [34, 35]. The double trace deformation can be non-local in the dual field theory, generically breaking Lorentz invariance. We will give a concrete proposal for this deformation that matches with our results for the singularity structure of the correlators.

Let us introduce in the field-theory Lagrangian a double trace deformation involving a scalar operator where the two insertions are separated by a relative constant time $T_0 > 0$:

$$S_g = \int d^d x \mathcal{L}_0 + g \int d^d x \mathcal{O}(t + T_0, \mathbf{x}) \mathcal{O}(t, \mathbf{x}), \quad (53)$$

where g is coupling of the double trace.

In order to compute the time-ordered correlation function in the presence of the double trace deformation, we will perform a formal expansion in g . For economy, we will suppress the spatial dependence of operators and keep explicitly only the time dependence

$$\langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_g = \langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_0 + g \int d^d x' \langle T\mathcal{O}(t)\mathcal{O}(0)\mathcal{O}(t' + T_0)\mathcal{O}(t') \rangle_0 + O(g^2). \quad (54)$$

Let us take advantage of the large- N limit and treat \mathcal{O} as an essentially free field. The logic behind this is that the correlator above is not just the connected component but it has contributions from the factorization in correlators with a smaller number of insertions. In the large- N limit the connected four-point correlator is suppressed respect to these factorized contributions. Since the expectation value of the operator is zero, the leading

contribution comes from the factorization in two-point functions. Let us assume for the moment $t > t' + T_0 > t' > 0$, then

$$\langle T\mathcal{O}(t)\mathcal{O}(0)\mathcal{O}(t' + T_0)\mathcal{O}(t') \rangle_0 \simeq \langle T\mathcal{O}(t)\mathcal{O}(t' + T_0) \rangle_0 \langle T\mathcal{O}(t')\mathcal{O}(0) \rangle_0 + \dots \quad (55)$$

The dots involve a contraction of $\mathcal{O}(t)$ with $\mathcal{O}(0)$, so their contribution is just a one-loop renormalization (after integrating over x') of the overall factor of the two-point function in the absence of the deformation. The term we have written explicitly is more interesting, it involves propagation from 0 to t' and from $t' + T_0$ to t . The integration over t' in the full expression (54) suggests a convolution between propagators, however this is only possible if the two propagators connect at the same point. We can now take advantage of the Poincaré invariance of the undeformed theory, the propagator depends only on the difference in time, so we can shift the arguments $t' + T_0 \rightarrow t'$, $t \rightarrow t - T_0$, so the first propagator in (55) now connects t' to $t - T_0$. Now the two propagators connect at the same point at t' and after integrating in x' , we just have the convolution of the two. As usual, the convolution of two propagators is a propagator connecting the external points, so we get a propagator between 0 and $t - T_0$. We can repeat the argument for the remaining possibilities concerning time ordering, for instance if $0 > t' + T_0 > t' > t$, one propagator connects t to t' and the other $t' + T_0$ to 0. In this case we can shift $t \rightarrow t + T_0$ and $t' \rightarrow t' + T_0$ in order to make the convolution. Therefore,

$$\begin{aligned} \langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_g &= \mathcal{Z}_{\mathcal{O}} \langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_0 + \\ &g \int d^{d-1}x' \langle T\mathcal{O}(t - T_0)\mathcal{O}(0) \rangle_0 + g \int d^{d-1}x' \langle T\mathcal{O}(t + T_0)\mathcal{O}(0) \rangle_0 + O(g^2), \end{aligned} \quad (56)$$

where we have introduced a renormalization factor $\mathcal{Z}_{\mathcal{O}}$ to account for the other contributions.

The key point here is that if the two-point function of the undeformed theory has a singularity of the form

$$\langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_0 \sim \frac{1}{t^{2\Delta}}, \quad (57)$$

then, in the presence of the double trace deformation, there are new singularities that are displaced in time by $\pm T_0$

$$\langle T\mathcal{O}(t)\mathcal{O}(0) \rangle_g \sim \frac{1 + O(g)}{t^{2\Delta}} + \frac{g + O(g^2)}{(t - T_0)^{2\Delta}} + \frac{g + O(g^2)}{(t + T_0)^{2\Delta}} + O(g^2). \quad (58)$$

We can follow the same procedure to see the effect of $O(g^2)$ terms, they introduce singularities that are displaced in time by $\pm 2T_0$. In general, at order $O(g^n)$, new singularities

displaced by $\pm nT_0$ appear. Taking into account the full infinite sum, we find the same singularity structure as for the AdS/CFT setup with a static mirror sitting at $z = T_0$. This suggests that the non-local double trace deformation in (53) is the right one to describe Dirichlet boundary conditions in AdS. The straightforward generalization to a moving mirror would be to make T_0 a time-dependent function, given precisely by the trajectory of the mirror. From the field theory point of view this is a much more complicated problem, using AdS/CFT we can find a simple answer just applying geometric optics.

The analysis we have done is incomplete, since we have studied only two-point functions. In the presence of a mirror, higher point functions in the field theory side might be modified in a way that the double trace deformation does not capture. If that is the case, presumably higher order multi-trace deformations would be needed, the methods presented here may also be useful to determine them.

6 Spectral decomposition and scaling properties

To understand the physical implication of the change of amplitude we found in the case of the scaling mirror, it is useful to look at the spectrum of the field theory with non-local double-trace deformation. We start with the simple case of static mirror. The spectrum is given by the normalizable modes. In the WKB approximation, which contains the UV mode, is found in Appendix B to be discrete equal distant resonances: $\omega = \pm\omega_n \equiv \pm \frac{n\pi}{z_s}$. The corresponding spectral density is given by

$$\rho(\mu) \sim \sum_n \omega_n^d (\delta(\mu - \omega_n) - \delta(\mu + \omega_n)), \quad (59)$$

the power is fixed by dimension. A general two point correlator has the following representation:

$$\langle \mathcal{O}(t)\mathcal{O}(t') \rangle = \int_{-\infty}^{+\infty} d\omega d\mu \rho(\mu) \frac{e^{-i\omega(t-t')}}{\omega + \mu}, \quad (60)$$

The retarded and advanced correlators are obtained shifting the frequency by $\pm i\epsilon$, while the Wightman correlators are obtained by shifting the time. The time ordered correlator is a combination of those. Taking into account that the spectrum is discrete and applying the residue theorem in evaluating the integral, we obtain:

$$\langle \mathcal{O}(t)\mathcal{O}(t') \rangle = \sum_n \phi_n(t) \phi_n^*(t'), \quad (61)$$

with $\phi_n(t) \sim e^{-i\omega_n t} \omega_n^{\frac{d}{2}}$. With an equidistant spectrum, it is easy to check that $\phi_n(t - 2z_s) = \phi_n(t)$, which leads to the periodic behavior of the two-point correlator,

$$\langle \mathcal{O}(t - 2mz_s) \mathcal{O}(t') \rangle = \langle \mathcal{O}(t) \mathcal{O}(t') \rangle. \quad (62)$$

Therefore in the result of $\langle \mathcal{O}(t) \mathcal{O}(t') \rangle$, a singularity of the form $\frac{r}{(t-t')^{2c}}$ is always accompanied by singularities of the form $\frac{r}{(t-2mz_s-t')^{2c}}$, corresponding a periodic behavior of the returning signal.

Now we consider a falling mirror with scaling trajectory, we can still use (61) as a good representation of the correlator, but now $\phi_n(t) = t^{\lambda_n - c} \lambda_n^{\frac{d}{2}}$. $\lambda_n = \frac{2i\pi n}{\ln \frac{u_0+1}{u_0-1}}$ is obtained from the appendix of [31]. Similar to the case of a static mirror, we find:

$$\phi_n(t/\frac{u_0+1}{u_0-1}) = \left(\frac{u_0+1}{u_0-1}\right)^c \phi_n(t), \quad (63)$$

which leads to the identity for the correlator

$$\left\langle \mathcal{O}(t/\frac{u_0+1}{u_0-1}) \mathcal{O}(t') \right\rangle = \left(\frac{u_0+1}{u_0-1}\right)^c \langle \mathcal{O}(t) \mathcal{O}(t') \rangle. \quad (64)$$

If we expand the correlators appearing on both sides of (64) around the singularities:

$$\langle \mathcal{O}(t) \mathcal{O}(t') \rangle = \dots + \frac{r_0}{(t-t')^{2c}} + \frac{r_1}{\left(t - \frac{u_0+1}{u_0-1} t'\right)^{2c}} + \dots, \quad (65)$$

and

$$\left\langle \mathcal{O}(t/\frac{u_0+1}{u_0-1}) \mathcal{O}(t') \right\rangle = \dots + \frac{r_0}{\left(t/\frac{u_0+1}{u_0-1} - t'\right)^{2c}} + \frac{r_1}{\left(t/\frac{u_0+1}{u_0-1} - \frac{u_0+1}{u_0-1} t'\right)^{2c}} + \dots. \quad (66)$$

Then,

$$\left\langle \mathcal{O}(t/\frac{u_0+1}{u_0-1}) \mathcal{O}(t') \right\rangle = \left(\frac{u_0+1}{u_0-1}\right)^{2c} \left[\dots + \frac{r_0}{\left(t - \frac{u_0+1}{u_0-1} t'\right)^{2c}} + \frac{r_1}{\left(t - \left(\frac{u_0+1}{u_0-1}\right)^2 t'\right)^{2c}} + \dots \right] \quad (67)$$

Comparing with (64), the equality is true if

$$r_1 = \left(\frac{u_0+1}{u_0-1}\right)^c r_0 \quad (68)$$

or in general

$$r_{m+1} = \left(\frac{u_0+1}{u_0-1}\right)^c r_m \quad (69)$$

which agrees with our previous analysis.

7 Conclusion and Outlook

We have established a general procedure for computing the singularity structure of spatially integrated unequal-time correlators. The procedure is applicable to a bulk scalar in AdS space subject to a Dirichlet boundary condition along arbitrary time-like trajectories. The Dirichlet boundary condition is interpreted as non-local double trace deformation to the dual field theory, which generically leads to the emergence of new singularities.

An immediate application of the current procedure is to the gravitational collapse model of thermalization. Although the current procedure is formulated in a pure AdS background, it should be readily extendable to the case of thermal AdS ^{#3}. This can then be used to study the unequal-time two-point correlator in the far-from-equilibrium regime in the gravitational collapse model. Similar patterns of singularities are expected in the resulting two-point correlator. The separation and magnitude of the singularities contains valuable information on the temporal decoherence of the gauge fields in the thermalization process. These results will be complementary to the study of equal-time correlators as for instance considered in [21], and will provide concrete information useful for understanding the physics of thermalization in strongly coupled gauge theory.

Recently, two interesting phenomena have been found in the context of global AdS, which is dual to CFT on a sphere. The first one is the existence of undamped oscillation modes in a thermal state [36]. It implies that certain modes will never actually thermalize in a CFT on a sphere. It was argued that the existence of the oscillation modes originates from the evenly spaced spectrum of the corresponding operator. Our two examples have shown similar behavior: the modes for the static mirror (ω_n) and for the scaling mirror (λ_n) are evenly spaced, and we have found that singularities at arbitrary late time appear periodically in the first case and quasi-periodically in the second case. It would be interesting to see whether this persists in a realistic model of thermalization. The second phenomenon is the non-perturbative instability of global AdS due to the evenly spaced spectrum [37]. While our model is formulated in the Poincaré patch, the mirror provides another boundary, which effectively produces a confining box in the Poincaré patch. The non-perturbative instability of global AdS is due to the resonant coupling between modes of different frequencies, which is possible because they have simple ratios. In this respect, the model we consider is similar in that respect to global AdS, so in principle the same kind of instability could be present in the case of a static mirror. It will be interesting to explore this possibility.

^{#3}Some complications due to using thermal AdS propagators are possible, however the calculations will be simplified by using spatially integrated propagators.

Another observation for the moving mirror is that the form of the correlator in the scaling mirror is similar to the correlators found in ageing systems (for a review see [38]) that describe some out-of-equilibrium states with slow dynamics in condensed matter, like for instance a glass. In those systems, two-point functions do not only depend on the time difference $t - t'$ between the two insertion points, but also have a power-like dependence on t/t' . For this reason the number of scaling exponents necessary to fully determine the system is larger than in the usual equilibrium states. In the holographic model presented here, we also observe a power-like dependence on the initial time, and the exponents for $t - t'$ and t/t' are fixed by the dimension of the dual operator, at least in the massless case. It would be interesting to explore how far this connection between ageing systems and holography can reach.

Finally, it is worth stressing that the results obtained in this work rely on the large N_c limit. The relaxation of this limit might be important for thermalization in quantum field theory, as described for instance in [29, 39]. It is an interesting challenge to see how the finite N_c effect may change the results in this work.

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A Integrals involving $i\epsilon$

$$\begin{aligned}
\int_{-\infty}^{\infty} dy \frac{1}{(-iy + \epsilon)^{c+1}} \ln(y - x - i\epsilon') &= \frac{-2i\pi}{i^{c+1}} \frac{1}{(-x - i\epsilon'')^c} \\
\int_{-\infty}^{\infty} dy \frac{1}{(iy + \epsilon)^{c+1}} \ln(x - y - i\epsilon') &= \frac{-2i\pi}{i^{c+1}} \frac{1}{(x - i\epsilon'')^c} \\
\int_{-\infty}^{\infty} dy \frac{1}{(-iy + \epsilon)^{c+1}} \ln(x - y - i\epsilon') &= \int_{-\infty}^{\infty} dy \frac{1}{(iy + \epsilon)^{c+1}} \ln(y - x - i\epsilon') = 0 \\
\int_{-\infty}^{\infty} dy \frac{1}{(-iy + \epsilon)^{c+1}} \ln(-(x - y)^2 + i\epsilon') &= \frac{-2i\pi}{i^{c+1}} \frac{1}{(-x - i\epsilon'')^c} \\
\int_{-\infty}^{\infty} dy \frac{1}{(iy + \epsilon)^{c+1}} \ln(-(x - y)^2 + i\epsilon') &= \frac{-2i\pi}{i^{c+1}} \frac{1}{(x - i\epsilon'')^c}.
\end{aligned} \tag{70}$$

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{1}{(-iy + \epsilon)^{c+1}} \frac{1}{(x - y + i\epsilon)^c} &= (e^{-i\pi c} - e^{i\pi c}) i^{c+1} \frac{1}{(x + i\epsilon)^{2c}} B(2c, 1 - c) \\
\int_{-\infty}^{\infty} \frac{1}{(iy + \epsilon)^{c+1}} \frac{1}{(y - x + i\epsilon)^c} &= (e^{-i\pi c} - e^{i\pi c}) i^{c+1} \frac{1}{(-x + i\epsilon)^{2c}} B(2c, 1 - c) \\
\int_{-\infty}^{\infty} \frac{1}{(-iy + \epsilon)^{c+1}} \frac{1}{(y - x + i\epsilon)^c} &= 0 \\
\int_{-\infty}^{\infty} \frac{1}{(iy + \epsilon)^{c+1}} \frac{1}{(x - y + i\epsilon)^c} &= 0.
\end{aligned} \tag{71}$$

A few comments about these integrals are helpful: i) we have extended the integration range of y to infinity. This is justified as far as the singular part in x is concerned and the integral is convergent. It turns out the results of the integrals in (70) and (71) contain no regular part. If it were not the case, we should have discarded the regular parts; ii) The vanishing of the last two integrals is due to the fact that the branch cuts associated with the integrand can be chosen to lie at a single side of the half plane (upper or lower), thus the integrals are guaranteed to vanish by a proper choice of contour; iii) ϵ'' is a combination of ϵ and ϵ' , but the precise relation is not important, as we are interested in the limit $\epsilon \rightarrow 0$. We will simply omit the primes.

B Explicit evaluations of the correlators: static mirror

In order to compute the correlator, we write a general scalar field in the bulk in frequency space. Since we focus on the spatially integrated correlator, the scalar field depends on t and z only.

$$\phi(t, z) = \int g(\omega) \tilde{\phi}(\omega, z) e^{-i\omega \Delta t} d\omega, \tag{72}$$

where $\Delta t = t - t'$. $g(\omega)$ is the arbitrary Fourier coefficient and $\tilde{\phi}(\omega, z)$ satisfies the Laplacian equation in the bulk in the presence of a potential $V = V_0 \delta(z - z_s)$. $\tilde{\phi}(\omega, z)$ is easily solved by:

$$\tilde{\phi}(\omega, z) = z^{\frac{d}{2}} \left(A J_{\frac{d}{2}}(|\omega|z) + B J_{-\frac{d}{2}}(|\omega|z) \right), \quad z < z_s \tag{73}$$

$$\tilde{\phi}(\omega, z) = z^{\frac{d}{2}} C H_{\frac{d}{2}}^{(1)}(|\omega|z) \quad z > z_s. \tag{74}$$

The solutions above and below the potential are matched in a standard way:

$$\begin{cases} A J_{\frac{d}{2}}(|\omega|z_s) + B J_{-\frac{d}{2}}(|\omega|z_s) = C H_{\frac{d}{2}}^{(1)}(|\omega|z_s) \\ z_s^2 |\omega| \left[A J_{\frac{d}{2}-1}(|\omega|z_s) - B J_{-\frac{d}{2}+1}(|\omega|z_s) - C H_{\frac{d}{2}-1}^{(1)}(|\omega|z_s) \right] = V_0 C H_{\frac{d}{2}}^{(1)}(|\omega|z_s) \end{cases} \tag{75}$$

It is not difficult to convince ourselves that C is $\frac{1}{V_0}$ suppressed compared to A, B and to the LO in V_0 , we have

$$AJ_{\frac{d}{2}}(|\omega|z_s) + BJ_{-\frac{d}{2}}(|\omega|z_s) = 0. \quad (76)$$

We may take $A = 1$ and $B = \kappa(|\omega|) = -\frac{J_{\frac{d}{2}}(|\omega|z_s)}{J_{\frac{d}{2}}(|\omega|z_s)}$ for specifickness. Using the second line of (75), we obtain an explicit expression for C at order $\frac{1}{V_0}$:

$$C = \frac{z_s^2|\omega|}{V_0} \frac{J_{\frac{d}{2}-1}(|\omega|z_s) - \kappa(|\omega|)J_{-\frac{d}{2}+1}(|\omega|z_s)}{H_{\frac{d}{2}}^{(1)}(|\omega|z_s)}. \quad (77)$$

The scalar wave on the mirror is simply given by:

$$\begin{aligned} \tilde{\phi}(\omega, z = z_s) &= Cz^{\frac{d}{2}} H_{\frac{d}{2}}^{(1)}(|\omega|z_s) \\ &= \frac{z_s^{c+1}}{V_0} \sqrt{\frac{2|\omega|}{\pi}} \frac{\cos \frac{\pi(d-1)}{2}}{\cos \left(|\omega|z_s + \frac{\pi(d-1)}{4} \right)}, \end{aligned} \quad (78)$$

where in the last step, we have taken $|\omega| \rightarrow \infty$ to simplify the expression. This will not affect our results on the most singular part of G_{LO} , which is supposed to come from the UV physics. To calculate G_{LO} , we notice that $\phi(t, z)$ has the following expansion:

$$\phi(t, z) = \phi_0(t) + \cdots \phi_d(t)z^d + \cdots, \quad (79)$$

where the coefficients ϕ_0 and ϕ_d can be expressed by

$$\phi_0(\Delta t) = \int g(\omega) \kappa(\omega) \frac{1}{\Gamma(1 - \frac{d}{2})} \left(\frac{2}{|\omega|} \right)^{d/2} e^{-i\omega\Delta t} d\omega \quad (80)$$

$$\phi_d(\Delta t) = \int g(\omega) \frac{1}{\Gamma(1 + \frac{d}{2})} \left(\frac{|\omega|}{2} \right)^{d/2} e^{-i\omega\Delta t} d\omega. \quad (81)$$

On the other hand,

$$\phi(\Delta t, z = z_s) = \int g(\omega) C z^{\frac{d}{2}} H_{\frac{d}{2}}^{(1)}(|\omega|z_s) e^{-i\omega\Delta t} d\omega. \quad (82)$$

An easy way to calculate G_{LO} is to set $\phi_0(\Delta t) = \delta(\Delta t)$, which allows us to solve for $g(\omega)$. Plugging $g(\omega)$ to (82), we obtain $G_{LO}(\Delta t)$, in the following frequency space:

$$G_{LO}(\Delta t) = \int \frac{1}{2\pi} \frac{\sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}}} \frac{|\omega|^c z_s^{c+1} \cos \frac{\pi(d-1)}{2}}{\cos \left(|\omega|z_s - \frac{\pi(d+1)}{4} \right)} e^{-i\omega\Delta t} d\omega \quad (83)$$

We use the residue theorem to evaluate (83)^{#4}. The poles of the integrand is given by the zeros of $\cos(|\omega|z_s - \frac{\pi(d+1)}{4})$, which are located at $\omega = \pm\omega_n$, with $\omega_n = \frac{(n+\frac{d+3}{4})\pi}{z_s}$ ($n = 0, 1, \dots$). The poles lie along the real axis symmetrically. We need to deform the integration contour to avoid the poles. The time-ordered correlator can be obtained with the contour shifted slightly counter-clockwise. This corresponds to the following substitution:

$$\omega \rightarrow \omega(1 + i\epsilon), \quad t \rightarrow t(1 - i\epsilon). \quad (84)$$

For $\Delta t < 0$, we close the integration contour upwards. The poles on the negative real axis contribute. The residue come from $\frac{1}{\cos(|\omega|z_s - \frac{\pi(d+1)}{4})}$ are given by:

$$\text{res} \left(\frac{1}{\cos(|\omega|z_s - \frac{\pi(d+1)}{4})} \right) = \frac{(-1)^n}{z_s}. \quad (85)$$

Summing over the residues, we obtain

$$\begin{aligned} G_{LO}(\Delta t) &= \frac{i\pi^c \sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}}} \cos \frac{\pi(d-1)}{2} e^{\frac{i\pi(d+3)\Delta t}{4z_s}} (n + \frac{d+3}{4})^c e^{in\pi(\Delta t/z_s - 1) - \epsilon} \\ &= \frac{i\pi^c \sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}}} \cos \frac{\pi(d-1)}{2} e^{\frac{i\pi(d+3)\Delta t}{4z_s}} \Phi(w, s, v), \end{aligned} \quad (86)$$

where $w = e^{i\pi(\Delta t/z_s - 1) - \epsilon}$, $s = -c$ and $v = \frac{d+3}{4}$. $\Phi(w, s, v)$ is the Lerch transcendent function. the most singular part of G_{LO} follows from an expansion of the Lerch function:

$$\Phi(w, s, v) \stackrel{d}{=} \Gamma(1+c) e^{-\frac{i\pi(\Delta t - (2m+1)z_s)}{z_s} \frac{d+3}{4}} \left(-i\pi \frac{\Delta t - (2m+1)z_s}{z_s} \right)^{-c-1}. \quad (87)$$

In the limit $t = t_m(1+x)$, $x \rightarrow 0$, we obtain the most singular part of G_{LO} as

$$G_{LO}(\Delta t) = \frac{i\sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}} \pi} \cos \frac{\pi(d-1)}{2} e^{\frac{i\pi(1+2m)}{2}} \frac{1}{(-ix + \epsilon)^{c+1}} \left(\frac{z_s}{t_m} \right)^{c+1}. \quad (88)$$

For $\Delta t > 0$, we close the contour downwards and include contributions from poles on the positive real axis. We obtain the following expression for G_{LO} , after summing over the residues:

$$\begin{aligned} G_{LO}(\Delta t) &= \frac{i\pi^c \sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}}} \cos \frac{\pi(d-1)}{2} e^{-\frac{i\pi(d+3)\Delta t}{4z_s}} (n + \frac{d+3}{4})^c e^{-in\pi(\Delta t/z_s - 1) - \epsilon} \\ &= \frac{i\pi^c \sqrt{\frac{2}{\pi}} \Gamma(1 - \frac{d}{2})}{2^{\frac{d}{2}}} \cos \frac{\pi(d-1)}{2} e^{-\frac{i\pi(d+3)\Delta t}{4z_s}} \Phi(w, s, v), \end{aligned} \quad (89)$$

^{#4}The residue theorem is not directly applicable in the presence of $|\omega|$. A proper way to treat it is to make the substitution $|\omega| \rightarrow \sqrt{\omega^2 + \eta^2}$ and consider the limit $\eta \rightarrow 0$. The square root brings in branch cuts, but one can show the contribution from the contour wrapping the branch cuts vanishes. As a net result, we may focus on the isolated poles only

with $w = e^{-i\pi(\Delta t/z_s-1)-\epsilon}$, $s = -c$ and $v = \frac{d+3}{4}$. The expansion of the Lerch function gives the following most singular part of G_{LO} :

$$G_{LO}(\Delta t) = \frac{i\sqrt{\frac{2}{\pi}}\Gamma(1-\frac{d}{2})}{2^{\frac{d}{2}}\pi} \cos \frac{\pi(d-1)}{2} e^{-\frac{i\pi(1+2m)}{2}} \frac{1}{(ix+\epsilon)^{c+1}} \left(\frac{z_s}{t_m}\right)^{c+1}. \quad (90)$$

We can compare (88) and (90) with the results obtained from the recursion equations. The results are in perfect agreement.

We can further compute the two-point correlator and compare the results with the output from recursion equations. The evaluation of the two-point correlator closely resembles the evaluation of G_{LO} . It simply amounts to the evaluation of the following integral:

$$G(\Delta t) = \int \frac{1}{2\pi} \frac{1}{\kappa(\omega)} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(1+\frac{d}{2})} \left(\frac{|\omega|}{2}\right)^d e^{-i\omega\Delta t} d\omega. \quad (91)$$

We will not repeat the steps in doing the integral, but just state the final result. For $\Delta t < 0$, we have

$$G(\Delta t < 0) = -\frac{i \cos \frac{d+1}{2}\pi}{z_s} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(1+\frac{d}{2})} \left(\frac{\pi}{2z_s}\right)^d e^{\frac{i\frac{d+3}{4}\pi\Delta t(1-i\epsilon)}{z_s}} \Phi(w, s, v), \quad (92)$$

where $w = e^{\frac{i\pi\Delta t(1-i\epsilon)}{z_s}} = e^{\frac{i\pi\Delta t}{z_s}-\epsilon}$, $v = \frac{d+3}{4}$, $s = -d$. where we have used the same definition as before x : $t = \bar{t}_m(1+x)$ and $\bar{t}_m = t' + 2mz_s$. The most singular part of the correlator is given by

$$G(\Delta t < 0) \stackrel{d}{=} -i \cos \frac{d+1}{2}\pi \frac{\Gamma(1-\frac{d}{2})}{\Gamma(1+\frac{d}{2})} \left(\frac{\pi}{2}\right)^d \Gamma(1+d) e^{im\pi\frac{d+3}{2}} \frac{1}{(e^{-i\pi/2\pi\bar{t}_m})^{2c}} \frac{1}{(x+i\epsilon)^{2c}}. \quad (93)$$

For $\Delta t > 0$, we have

$$G(\Delta t < 0) = -\frac{i \cos \frac{d+1}{2}\pi}{z_s} \frac{\Gamma(1-\frac{d}{2})}{\Gamma(1+\frac{d}{2})} \left(\frac{\pi}{2z_s}\right)^d e^{-\frac{i\frac{d+3}{4}\pi\Delta t(1-i\epsilon)}{z_s}} \Phi(w, s, v), \quad (94)$$

where $w = e^{-\frac{i\pi\Delta t(1-i\epsilon)}{z_s}} = e^{-\frac{i\pi\Delta t}{z_s}-\epsilon}$, $v = \frac{d+3}{4}$, $s = -d$. The most singular part of the correlator is given by

$$G(\Delta t > 0) \stackrel{d}{=} -i \cos \frac{d+1}{2}\pi \frac{\Gamma(1-\frac{d}{2})}{\Gamma(1+\frac{d}{2})} \left(\frac{\pi}{2}\right)^d \Gamma(1+d) e^{-im\pi\frac{d+3}{2}} \frac{1}{(e^{-i\pi/2\pi\bar{t}_m})^{2c}} \frac{1}{(-x+i\epsilon)^{2c}}. \quad (95)$$

We can verify that (93), (95) and (36) again agree with each other!

C Explicit evaluations of the correlators: scaling trajectory

The scaling trajectory corresponds to the potential $V = V_0 \delta(\frac{t}{z} - u_0)$. As familiar from quantum mechanics, the solution to the wave equation in the bulk is solved by a matching of the wave above and below the mirror. We closely follow the notation of [31]. For the eigenmode $\phi(u, v) = v^{\frac{\lambda}{2}} f(u)$, the incoming/outgoing solutions are given by:

$$f_{\pm}(u) = u^{-\frac{\lambda}{2} - \frac{d-1}{4}} (u \mp 1)^{\lambda} F\left(\frac{1-d}{2}, \frac{1+d}{2}; \pm\lambda + 1, \frac{1-u}{2}\right). \quad (96)$$

Above the mirror, the solution is a combination of incoming and outgoing waves $f(u) = Af_+(u) + Bf_-(u)$ and below the mirror there is only incoming component $f(u) = Cf_+(u)$ ^{#5}. Matching on the mirror gives:

$$\begin{cases} Af'_+(u_0) + Bf'_-(u_0) = Cf'_+(u_0) + \frac{V_0}{u_0^2-1} Cf_+(u_0) \\ Af_+(u_0) + Bf_-(u_0) = Cf_+(u_0) \end{cases} \quad (97)$$

We choose the normalization such that $A, B \sim 1, C \sim \frac{1}{V_0}$ as $V_0 \rightarrow \infty$. We work to the LO in V_0 from now on. We choose

$$\begin{aligned} A &= (u_0 + 1)^{\lambda} F\left(\frac{1-d}{2}, \frac{1+d}{2}; -\lambda + 1, \frac{1-u_0}{2}\right) \\ B &= -(u_0 - 1)^{\lambda} F\left(\frac{1-d}{2}, \frac{1+d}{2}; \lambda + 1, \frac{1-u_0}{2}\right), \end{aligned}$$

according to $Af_+(u_0) + Bf_-(u_0) = 0$. It follows as $\lambda \rightarrow \infty$,

$$C = \frac{u_0^2 - 1}{V_0} \frac{Af'_+(u_0) + Bf'_-(u_0)}{f_+(u_0)} \rightarrow \frac{2\lambda(u_0 + 1)^{\lambda}}{V_0}. \quad (98)$$

The bulk-boundary propagator is built as follows:

$$\begin{aligned} \delta(t - t') &= \phi^0(t) = \int K(\lambda, A, B) t^{\lambda + \frac{d-1}{2}} g(\lambda) d\lambda \\ G(t, z, t') &= \int C \phi_{\lambda}(t, z) g(\lambda) d\lambda \\ \Rightarrow G(t, z, t') &= \frac{1}{2\pi i} \int d\lambda \frac{C \phi_{\lambda}(t, z)}{K(\lambda, A, B)} t'^{-\lambda - \frac{d+1}{2}}. \end{aligned} \quad (99)$$

We first compute $G(t, z, t')$ along the mirror trajectory, that is we set $z = \frac{t}{u_0}$. As $\lambda \rightarrow \infty$, (99) leads to the following representation of G_{LO} :

$$G_{LO}(t, t') \rightarrow \frac{A_1}{2\pi i} \int \frac{\lambda(u_0 + 1)^{\lambda} \left(1 - \frac{1}{u_0}\right)^{\lambda}}{\frac{\Gamma(\lambda+1)}{\Gamma(\lambda + \frac{d+1}{2})} (u_0 + 1)^{\lambda} - \frac{\Gamma(-\lambda+1)}{\Gamma(-\lambda + \frac{d+1}{2})} (u_0 - 1)^{\lambda}} \left(\frac{t}{t'}\right)^{\lambda} t'^{-\frac{d+1}{2}} \left(\frac{t}{u_0}\right)^{\frac{d-1}{2}} d\lambda, \quad (100)$$

^{#5}The time-ordered correlator actually requires incoming wave for $\frac{\lambda}{t} > 0$ and outgoing wave for $\frac{\lambda}{t} < 0$. However, it is a short exercise to show that either incoming or outgoing wave below the mirror gives rise to the same $\phi_{\lambda}(t, z = t/u_0)$, thus the following computation is not affected.

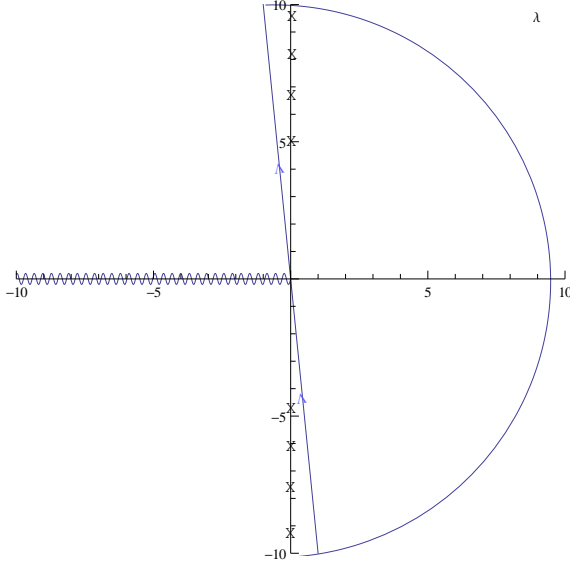


Figure 2: (color online) The isolated singularities lie symmetrically along the imaginary axis. A branch cut extends from the origin to negative infinity. The integration contour is shifted slightly counter-clockwise for the time-ordered correlator.

with $A_1 = 2^{\frac{d+1}{2}} \frac{\Gamma(\frac{d+1}{2})}{\Gamma(d)}$. The integral in (100) is not well defined. Certain $i\epsilon$ prescription is needed. Since we are interested in the time-ordered correlator, we deform the integration contour as follows: $\lambda \rightarrow \lambda(1 + i\epsilon)$, together with $\ln \frac{t}{t'} \rightarrow \ln \frac{t}{t'}(1 - i\epsilon)$, to ensure the reality of the Fourier factor $(\frac{t}{t'})^\lambda$. The integration contour is shown in Fig.2.

For $t < t'$, we can close the contour on the right half plane to include residues on the positive imaginary axis. Similar to the appendix of [31], we have:

$$G_{LO}(t < t') = \frac{A_1}{a} t'^{-\frac{d+1}{2}} \left(\frac{t}{u_0} \right)^{\frac{d-1}{2}} e^{\frac{i\pi b(d-1)}{2a}} \left(\frac{2i\pi}{a} \right)^c \Phi(e^{\frac{2i\pi b}{a}}, -c, \frac{d-1}{4}), \quad (101)$$

with $a = \ln \frac{u_0+1}{u_0-1}$, $b = \ln \frac{t}{t'} + \ln \frac{u_0-1}{u_0}$, $c = \frac{d+1}{2}$. Φ is the Lerch transcendent function. The $i\epsilon$ prescription above gives $b \rightarrow b + i\epsilon$. It is important for fixing the ambiguity in the argument of the Lerch transcendent function. Using the following decomposition formula for Lerch transcendent:

$$\Phi(w, s, v) = \Gamma(1-s)w^{-v} \left(\ln \frac{1}{w} \right)^{s-1} + w^{-v} \sum_{r=0}^{\infty} \zeta(s-r, v) \frac{(\ln w)^r}{r!} \quad (102)$$

Define $t_n = t' \frac{u_0}{u_0-1} \left(\frac{u_0+1}{u_0-1} \right)^n$. It is easy to see t_n are just the points where the bouncing light ray hit the mirror. As $t \rightarrow t_n$, $\ln w \rightarrow 0$, the first term in (102) is singular while the sum is

regular. We obtain the singular part of $G_{LO}(t < t')$ given by:

$$G_{LO}(t < t') \stackrel{d}{=} \frac{A_1}{a^{c+1}} t'^{-\frac{d+1}{2}} \left(\frac{t}{u_0} \right)^{\frac{d-1}{2}} e^{\frac{i\pi(d-1)n}{2}} \frac{\Gamma(1+c)}{2\pi} \frac{i^c}{(-if + \epsilon)^{c+1}}, \quad (103)$$

where $f = \frac{b}{a} - n = \frac{\ln \frac{t}{t_n}}{a}$, $|f| < \frac{1}{2}$, $n < 0$. The symbol implies that the equality holds as far as the singular part is concerned. In writing i^c , we have chosen $\arg(\pm i) = \pm \frac{\pi}{2}$ to simplify the notation.

For $t > t'$, we make a change of variable $\lambda \rightarrow -\lambda$ in (100). The latter now becomes:

$$G_{LO}(t > t') \rightarrow \frac{A_1}{2\pi i} \int \frac{-\lambda u_0^\lambda}{\frac{\Gamma(-\lambda+1)}{\Gamma(-\lambda+\frac{d+1}{2})}(u_0-1)^\lambda - \frac{\Gamma(\lambda+1)}{\Gamma(\lambda+\frac{d+1}{2})}(u_0+1)^\lambda} \left(\frac{t'}{t} \right)^\lambda t'^{-\frac{d+1}{2}} \left(\frac{t}{u_0} \right)^{\frac{d-1}{2}} d\lambda \quad (104)$$

We can again close the contour on the right half plane to obtain:

$$G_{LO}(t > t') \stackrel{d}{=} \frac{A_1}{a^{c+1}} t'^{-\frac{d+1}{2}} \left(\frac{t}{u_0} \right)^{\frac{d-1}{2}} e^{-\frac{i\pi(d-1)(n+1)}{2}} \frac{\Gamma(1+c)}{2\pi} \frac{i^c}{(if + \epsilon)^{c+1}}, \quad (105)$$

with $f = \frac{\ln \frac{t}{t_n}}{a}$, $|f| < \frac{1}{2}$, $n \geq 0$.

Define y as $t = t_n(1+y)$. The dimensionless ratio y measured the closeness of t to t_n . To the LO in y , $\ln \frac{t}{t_n} \stackrel{d}{=} y$, thus we obtain the most singular part of $G(t, t')$ as follows:

$$G_{LO}(t < t') \stackrel{d}{=} A_2 e^{\frac{i\pi(d-1)n}{2}} \frac{1}{(-iy + \epsilon)^{c+1}} \quad n < 0 \quad (106)$$

$$G_{LO}(t > t') \stackrel{d}{=} A_2 e^{\frac{-i\pi(d-1)(n+1)}{2}} \frac{1}{(iy + \epsilon)^{c+1}} \quad n \geq 0, \quad (107)$$

with $A_2 = A_1 t'^{-\frac{d+1}{2}} \left(\frac{t_n}{u_0} \right)^{\frac{d-1}{2}} \frac{\Gamma(1+c)}{2\pi} i^c$.

We also want to compute the two-point correlator by direct evaluation of the λ -integral in Mellin representation. It is not difficult to see that the time-ordered correlator for $t > t'$ and $t < t'$ are basically the contribution corresponding to the lower sign and upper sign respectively in the retarded correlator in (3.19) of [31]. The most singular parts are given by the following:

$$\begin{aligned} t > t' : & T\langle O(t = \bar{t}_m(1+x)) O(t') \rangle \\ & \stackrel{d}{=} (\bar{t}_m t')^{-\frac{d+1}{2}} 2^d (1 - e^{i\pi d}) \frac{\Gamma(-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{1-d}{2})\Gamma(d)} e^{\frac{-i\pi(d-1)m}{2}} \frac{\Gamma(1+d)}{e^{-i\pi(d+1)}(-x+i\epsilon)^{d+1}2i\pi} \end{aligned} \quad (108)$$

$$\begin{aligned} t < t' : & T\langle O(t = \bar{t}_m(1+x)) O(t') \rangle \\ & \stackrel{d}{=} (\bar{t}_m t')^{-\frac{d+1}{2}} 2^d (1 - e^{-i\pi d}) \frac{\Gamma(-d)\Gamma(\frac{1+d}{2})}{\Gamma(\frac{1-d}{2})\Gamma(d)} e^{\frac{i\pi(d-1)m}{2}} \frac{\Gamma(1+d)}{e^{-i\pi(d+1)}(x+i\epsilon)^{d+1}2i\pi}. \end{aligned} \quad (109)$$

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